

# Reliability and risk analysis of micropile bearing capacity based on SPT variability: Case Study

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**ABSTRACT:** This paper evaluates the reliability index of a single micropile ( $\phi = 0.31$  m and  $L = 16$  m) under axial loading based on the variability of eighth Standard Penetration Tests used as site characterization for a underpinning solution of a viaduct in the southern region of the State of Rio de Janeiro, Brazil. Horizons of varied granulometry ranging from clays to sand, with high consistencies or compactness were identified in such tests. Two probabilistic approaches were performed in order to obtain the reliability index ( $\beta$ ): First-Order Second-Moment Method (FOSM) and Monte Carlo Method (MCM) with 100.000 simulations. As a result, it was possible to analyze the probability of failure of the foundation whereas it is dependent on the reliability index. Furthermore, six hypothetical scenarios of different pile lengths (from 15 to 10 meters) were evaluated in order to analyse if the performed pile length was conservative based on technical literature. It was possible to verify that the performed micropile ( $\phi = 0.31$  m and  $L = 16$  m) presented conservative values of reliability index and the optimum length of the pile should be between 13 and 15 meters for a cost-effective design. Finally, a mixed load test was conducted up to twice the workload in order to verify the performance of the micropile and as realibility analysis validation. Geotechnical failure was not evidenced in the test and a maximum displacement of 2.8 mm was reached with a load of 1544 kN.

**Keywords:** Micropile; Underpinning; SPT; Reliability; Failure Probability.

## 1. Introduction

Micropile foundation are often used as underpinning solution to support additional structural load without excessive vibrations. The concept of micropiles was developed by professor Lizzi which consists in small-pile structures, drilled and grouted with or without pressure [1].

The construction process influences the behavior of micropiles, specifically the grouting method. The presence or not of pressure grout and its magnitude influences the type of micropile [2].

Among the types specified by [2], type-B micropiles or root piles are the most used in Brazil. Grout injection is performed from the bottom up as the temporary drill casing is withdrawn in order to fill the pile bore hole. Immediately after the shaft been formed, pressure is applied at the top of the pile with compressed air range from 0.5 to 1 MPa, one or more times, during removal of the casing tube. This technique aims to improve shaft resistance and reduce shaft imperfection.

In terms of geotechnical capacity, it is widely assumed that the vertical micropile resistance under axial loading is developed mainly by friction or adhesion along the shaft. This behavior is justified because two basic boundary conditions: small cross section area in comparison with the pile length and debris deposit at the bottom of the tip. The last condition it's due to the accumulation of soft soil resulted of the installation technique [3].

It is noteworthy that if the micropile is performed embedded in rock without debris under the pile, tip resistance may contribute significantly to the overall capacity of the micropile [3].

As any other type of pile foundation performed to support important structures, it is important to analyze whether its behavior is reliable or not. In general, the traditional procedure used in geotechnical pile design addresses deterministic methods. In other words, global or partial coefficients are applied to cover the overall uncertainties of the model.

The problem with this approach is that uncertainties derived from geotechnical investigations, spatial soil variability, pile executive process and calculation methods are included in a single factor (global coefficient) or divided in partial factors, which may lead to misconceptions about the safety of the foundation. In order to evaluate the variability of these parameters and the impact of them on the reliability of the structure, probabilistic methods can be used as a form to understand the impact of each uncertain in the design. Therefore, in the last two decades, reliability methods have become increasingly used by geotechnical engineers as a tool for assessing safety and mitigating future accidents [4, 5, 6, 7, 8].

The first purpose of this paper is to demonstrate the application of two reliability methods in a case study to evaluate the safety condition of a single micropile foundation under axial loading.

The reliability based-design (RBD) approaches conducted in this study, the first-order reliability method (FOSM) and Monte Carlo method (MCM), were applied in a performance function of pile bearing capacity with

two uncertainties: the variability of N value obtained from SPT along the depth and the variability of the load demand. Thus, the ultimate limit state of the pile foundation will be analyzed.

Another purpose of this paper is to verify whether the optimum pile length was considered based on current standards and technical literature or not.

## 2. Reliability approach

### 2.1. Basic Concepts

There are several ways to measure uncertainties linked to large engineering projects, and therefore calculate probability of success or failure. The level of accuracy of the reliability methods depends basically on two fundamental points: the complexity of the limit state functions, and the number of uncertainties involved in the process.

The limit state function or performance function will define whether the structure may fail or not. For example, in structural reliability it can be stated that a given structural element will fail when its load-carry capacity is insufficient to support the load-effects (dead load, live load, wind effects). In this case the performance function will be considered an Ultimate Limit State (ULS) function. If the structure fails to provide enough resistance to avoid gradual deterioration, excess of deformation or vibration, the limiting function governing this phenomenon will be considered a Serviceability Limit State (SLS) function [9].

This paper will focus on the study of the Ultimate Limit State of single micropile under axial load.

### 2.2. General approach

FOSM and MSM procedures are based on the following general methodology:

1. Definition of the performance function governing the phenomenon ( $g(X_i)$ ). Generally the limit state function is defined based on resistance and demand as in Eq.(1):

$$g(R, D) = R - D = g(X_i) \quad (1)$$

where  $g(R, D)$  is the safety margin or limit boundary,  $R$  denotes the resistance,  $D$  represents the demand and  $X_i$  are the random variables;

- If  $g > 0 \rightarrow$  safe structure;
  - If  $g = 0 \rightarrow$  limit zone between safe and unsafe;
  - If  $g \leq 0 \rightarrow$  unsafe structure;
2. Identification of the random and deterministic variables to be considered ( $X_i$ );
  3. Description and characterization of the variables as statistical parameters: mean ( $\mu$ ), standard deviation (SD), coefficient of variation (CV) and distribution types (probability density function – PDF).
  4. Identification of the type of distribution of each variable involved in the process: uniform, normal, lognormal, gamma,

Gumbel, as well as of the dependencies among them (by using COV – covariance matrix);

5. Calculate the reliability index ( $\beta$ ) or probability of failure ( $p_f$ ) based on the following relationship Eq.(2):

$$p_f = \Phi(-\beta) = 1 - \Phi(\beta) \quad (2)$$

where  $\Phi$  is the normal cumulative density function with mean 0 and variance 1;

6. Considering  $R$  and  $D$  as random variables, both of them have a PDF that characterize their behaviour. Therefore, the failure probability can also be expressed graphically as shown in Fig. 1 (hypothetical figure).

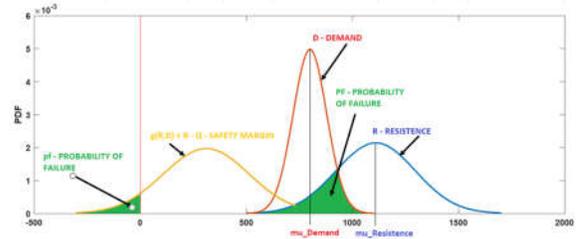


Figure 1. PDFs of demand, resistance and safety margin.

7. Another way to visualize safe and unsafe zones is to represent variables in a space domain. For example, by adopting the X-axis as a Resistance PDF, the Y-axis as a Demand PDF and applying the limit state function, the result will be a three-dimensional representation of the joint density function (Fig. 2). One more time, the limit state function separates the safe and unsafe domains.

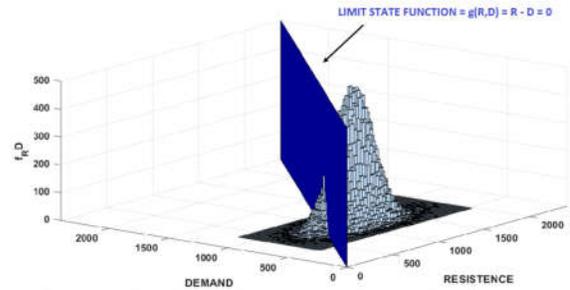


Figure 2. Three-dimensional visualization of a random joint density function  $f_{RD}$

A probability scale was proposed by [10] to relate the reliability index to the failure occurrence and its description (Table 1).

Table 1. Probability scale proposed by Clemens (1983)

$\beta$	pf	Description
-7.94	1:1	Collapse
0.52	1:3	Frequent
1.88	1:33	Probable
2.75	1:336	Occasional
3.43	1:3334	Remote
4.53	1:3x10 <sup>5</sup>	Improbable
7.27	1:6x10 <sup>6</sup>	Never

In pile reliability design, values of reliability index ( $\beta$ ) between 2.5 and 3.0, corresponding to a ruin probability of  $1 \times 10^{-3}$ , point out to be ideal for the geotechnical design of single piles [11]. Also, according to these authors, as pile foundations are usually designed to work in groups, the failure of a single pile will not necessarily cause the group to fail. If the lowest bearing capacity pile starts to fail, the load will be redistributed to the other piles of the group and the foundation will remain within its Ultimate Limit State (ULS). Therefore, when it comes to piles working together as a group, a  $\beta$  between 2.0 and 2.5 can be adopted corresponding to a failure probability of  $1 \times 10^{-2}$ .

There are cases where the pile design is based only in deterministic values as the Brazilian pile design standard [12]. All uncertainties are included in a global factor of safety (FOS). This factor is the ratio of the mean value of the Resistance PDF to the mean value of the Demand PDF as shown in Fig. 1 and Eq. 3.

$$FOS = \frac{\mu_{Resistance}}{\mu_{Demand}} \quad (3)$$

According to [12], for deep foundations under compression axial loads, the overall FOS must be equal to 2.0. The specification also indicates an attenuation of the FOS value as a function of the number of load tests performed (it can be reduced from 2.0 to 1.6). However, the load tests must be performed before the construction of the pile, in other terms, during pile design phase project. The problem with this approach is that it does not consider the probability failure zone of the PDF's involved, which could lead to unsafe situations.

### 2.3. First-Order Second-Moment (FOSM)

To understand the concept of reliability index and to facilitate algebraic analysis it is necessary to parameterize the state limit function in reduced variables. This means to use the standard form of the random variables of the performance function  $g(X_i)$ .

Assuming the limit state function as a function of the resistance and demand random variables as previously described (Eq. 1), their standard form can be obtained by subtracting the value of each variable by the mean and then dividing the result by its standard deviation as stated on Eq. (4,5):

$$Z_R = \frac{R - \mu_R}{\sigma_R} \quad (4)$$

$$Z_D = \frac{D - \mu_D}{\sigma_D} \quad (5)$$

The resulted variables  $Z_R$  and  $Z_D$  can also be expressed as Eq. (5,6)

$$R = \mu_R + Z_R * \sigma_R \quad (5)$$

$$D = \mu_D + Z_D * \sigma_D \quad (6)$$

Therefore, the performance function  $g(R,D)$  can also be described as reduced variables ( $Z_R, Z_D$ ) Eq. (7):

$$g(Z_R, Z_D) = \mu_R + Z_R * \sigma_R - \mu_D - Z_D * \sigma_D \quad (7)$$

In essence, this means Eq. (7) describes a linear function represented by the reduced variables  $Z_R$  and  $Z_D$ . For reliability analysis the target line of interest is when the performance function is equal to zero. In other words, this line will define safe and unsafe boundary conditions [9].

Within the scope of the present discussion, [13] introduced the concept of reliability index as the shortest distance from the origin of the reduced variables graph (Fig. 3) to the line  $g(R,D) = 0$ .

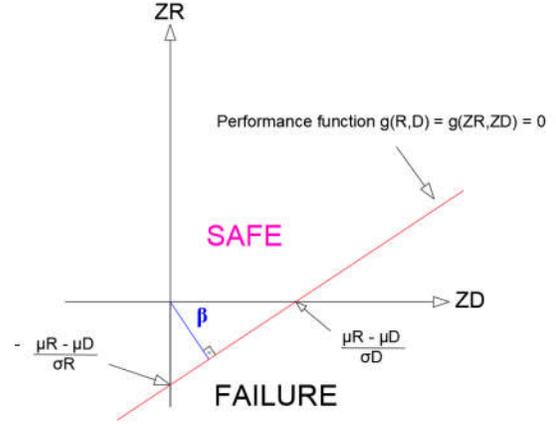


Figure 3. Reliability index defined as Hasofer and Lind (1974)

By geometric analysis it is possible to calculate the reliability index from Eq. (8):

$$\beta = \frac{\mu_R - \mu_D}{\sqrt{\sigma_R^2 + \sigma_D^2}} \quad (8)$$

where  $\beta$  is the inverse of the CV of the limit state function when the random variables are uncorrelated. It is noteworthy to state that for normally distributed random variables, the reliability index can be related with the failure probability and calculated as the Eq. (2).

For linear state functions  $g(X_i)$  with  $X_i$  random variables (Eq. 9, 10) the Hasofer-Lind reliability index can be obtained directly by the First-Order Second-Method methodology. This method simplifies the steps described previously in this section so that no graphing is required. The expression Eq. (11) express the FOSM reliability index for linear functions.

$$g(X_1, X_2, \dots, X_n) = a_0 + a_1 X_1 + a_2 X_2 + \dots + a_n X_n \quad (9)$$

$$g(X_1, X_2, \dots, X_n) = a_0 + \sum_{i=1}^n a_i * X_i \quad (10)$$

$$\beta = \frac{a_0 + \sum_{i=1}^n a_i * \mu_{x_i}}{\sqrt{\sum_{i=1}^n (a_i * \sigma_{x_i})^2}} \quad (11)$$

where  $a_i$  terms are constants,  $X_i$  are random uncorrelated variables,  $\mu_{x_i}$  and  $\sigma_{x_i}$  and are the mean values and the standard deviation values of the random variables, respectively.

It can be verified that the Hasofer-Lind reliability index depends only on the mean and standard deviation values of the considered variables. This is why the method was named the second-moment of the safety

analysis because only the first two moments (mean and variance) are needed for its calculation. Moreover, this method does not require previous knowledge of the type of probability distribution. However, if the random variables (RVs) are normally distributed and uncorrelated, this method obeys the relationship expressed in Eq. (2). Therefore, if the RVs do not follow this type of distribution, Eq. (11) only ensures an estimation of relating  $\beta$  with  $p_f$  [9,13].

One form to verify the normality of a distributed function is to apply Kolmogorov–Smirnov test (kstest). The one-sample kstest is a nonparametric test with the null hypothesis that the population CDF of the data is equal to the empirical CDF. In other words, the test verifies if there is a smooth fit between the distribution of a set of finite sample values and a standard normal CDF [14].

Finally, if the performance function is nonlinear, it can be linearized from a Taylor series expansion. This method will not be addressed in the present paper because the limit state function used to analyze the performance of the micropile bearing capacity behaves linearly.

#### 2.4. Monte Carlo Method (MCM)

The Monte Carlo Method is a simulation technique which is used to generate a numerical process of calculating the same expression repeatedly [15]. This expression can be a function of both deterministic and random variables or only a function of random variables, if the level of uncertainty involved is substantial. Therefore, the knowledge of the type of the distribution of each random variable used on the simulation before running it is indispensable. Moreover, it is also important to declare if the random variables are correlated or not.

The output data of the MSM is basically whether the numerical process of repeatedly calculating the test expression results in a condition stated before the simulation or not. For reliability analyses this condition will be whether the structure will fail or not which means its probability of failure ( $p_f$ ).

Hence, following the general steps (1–3) previously described, reliability analysis using MCM is performed as follows:

- Generate  $n$  (number of simulations) values for each variable based on their variability data (mean, standard deviation, coefficient of variation and PDF) and correlations if there is one;
- Calculate the value of the limit state function for each simulation;
- Determine the probability of failure as the sum of the simulations that fail ( $g(R,D)<0$ ) divided by the total number of simulations  $n$ , Eq.(12):

$$p_f = \frac{\text{number of times that } g(R,D) \leq 0}{\text{total number of simulations } n} \quad (12)$$

As well as in FOSM methodology, if the random variables are normally distributed and uncorrelated this method follows the relationship expressed in Eq. (2).

The MCM is a powerful tool to perform reliability analysis since it can address all types of random variables distributions (PDFs) on its simulations without distorting the reliability index value. On the other hand, the MCM restriction is related to the high computational costs. For complex problems a relevant computer machine is required to perform the simulations optimally.

For the case study considered in this paper, all MCM calculations were implemented using a routine in the software MATLAB, a matrix programming language and environment for statistical and graphical computation [16].

#### 2.5. Reliability analyses of axial loaded micropiles

To perform a proper design of any geotechnical structure, a prior soil investigation of the region where the structure will be implemented is required. The Standard Penetration Test (SPT) is the most used method throughout the world as a introductory field investigation, specially when the geotechnical structure to be addressed is a pile design [17]. Therefore, the  $N$  value of the SPT test is extensively used to predict the bearing capacity of piles [18, 19, 20, 21]. As a result, several pile design specifications worldwide adopt pile bearing capacity empirical expression based on  $N$ -SPT [12, 22, 23].

Due to SPT global popularity, the Standard Penetration In Situ Test was selected to estimate the vertical pile carrying capacity in the case described in this paper. Moreover, in Brazil, it is common to use empirical methods to calculate the load capacity of a foundation element. The most common ones were developed based on in situ tests, mainly SPT. Therefore, among the formulas employed in the world, the equation performed in this study will be the one proposed by a Brazilian geotechnical engineer [19, 20]. The consistency values obtained from Decourt's formula when compared to statistic load test results provides a smooth approximation of the bearing capacity of Brazilian tropical soils.

The basic equation of bearing capacity proposed by [19, 20] is (Eq. 13):

$$R_B = R_T + R_S \quad (13)$$

where  $R_T$  is the pile tip resistance (Eq. 14),  $R_S$  is the pile shaft resistance (Eq. 15) and  $R_B$  is the pile bearing capacity.

$$R_T = \alpha * C * N_T * A_T \quad (14)$$

where  $\alpha$  is the pile tip coefficient that depends on the type of the pile as well as the type of the soil in this region (Tab. 2),  $C$  is the characteristic resistance of the tip soil (Tab. 3),  $N_P$  is the average  $N$  value from the SPT at the tip or pile base, obtained from three values: the one corresponding to the tip level, the immediately preceding and the immediately after and  $A_P$  is the cross section area of the base.

**Table 2.**  $\alpha$  coefficient by Décourt (1996)

SOIL TYPE	PILE TYPE	
	TYPE-B MI-CROPILE	TYPE-D MI-CROPILE
Clay	0.85	1.0
Residual Soils/Intermediate Soils	0.6	1.0
Sand	0.5	1.0

**Table 3.** C coefficient by Décourt (1996)

SOIL TYPE	C (kPa)
Clay	120
Silty Clay	200
Silty Sand	250
Sand	400

$$R_S = \beta * 10 * \left(\frac{N_S}{3} + 1\right) * U * L \quad (15)$$

where  $\beta$  is the pile shaft coefficient that depends on the type of the pile as well as the type of the soil surrounding the pile shaft (Tab. 4),  $N_S$  is the average N value from the SPT along the pile shaft without the values used in the evaluation of the tip resistance ( $N_P$ ), U is the perimeter of the pile and L its length.

**Table 4.**  $\beta$  coefficient by Décourt (1996)

SOIL TYPE	PILE TYPE	
	TYPE-B MI-CROPILE	TYPE-D MI-CROPILE
Clay	1.5	3.0
Residual Soils/Intermediate Soils	1.5	3.0
Sand	1.5	3.0

Consequently, the limit state function, in a deterministic form, for axial load pile, is expressed as (Eq. 16):

$$g(X_i) = \alpha * C * N_T * A_T + \beta * 10 * \left(\frac{N_S}{3} + 1\right) * U * L - D \quad (16)$$

Some boundary conditions were applied due to the type of pile to be addressed in this study. First of all, as micropile usually does not mobilize tip resistance as previously stated, the pile tip resistance (RP) will be removed from the performance equation. Second of all, as it is usual to have site engineers controlling the operation of the piles, their dimensions (length and area) will be considered as deterministic and with low impact in the overall uncertainty. Finally the performance function to be used in this paper is indicated as (Eq. 17):

$$g(X_i) = g(N_S, S) = \beta * 10 * \left(\frac{N_S}{3} + 1\right) * U * L - D \quad (17)$$

where  $N_S$  and D (demand) will be considered as random variables and  $\beta$ , U and L constants.

### 3. Description of the case study

This case pertains to an expansion of a viaduct located in the south region of the State of Rio de Janeiro (SRJ),

bordering the State of São Paulo (SSP), Brazil. The viaduct, initially constructed over spread and caisson foundations, was extended almost symmetrically to both sides due to the increase of the traffic volume in the region.

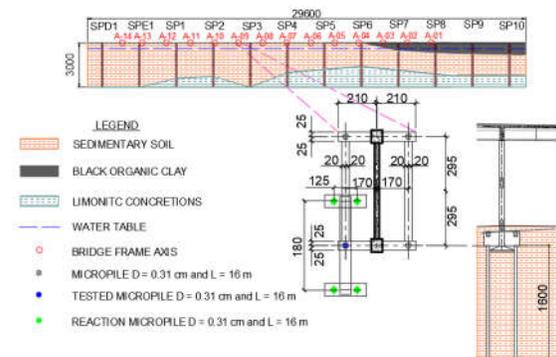
In order to dissipate the extra loads from the viaduct expansion into the ground, micropiles were designed and performed to underpin the prior foundation.

For the foundation underpinning design project, a geotechnical investigation campaign was carried out consisting of twelve Standard Penetration Test (SPD1, SPE1, SP1, ..., SP10). It was verified that the subsoil profile consisted of a region of tertiary sediments of varying particle size, from clays to sands, with high consistency and compactness. From the center of the viaduct towards SSP, peaks of resistance were observed due to the presence of limonitic concretions (SPD1, SPE1, SP1 to SP6). This panorama also describes the region corresponding to the center of the viaduct towards the SRJ. Moreover, the presence of horizons of tertiary soils towards SRJ is deeper, and upon them were found colluvial and even alluvial soils, in the form of soft, black organic and compressible clays (SP7 to SP10).

For a better analysis of the sections of the project, the viaduct was divided into frame axis (1 to 16) starting from the SSP towards SRJ. Each frame axis correspond to a structure formed by a pair of piers that support the slabs above them. The loads provinent of these structure are transferred to the foundations elements (old foundations and micropiles reinforcement) and then to the ground (Fig. 4). A summary of the number of micropiles to underpinn each frame axis of the viaduct and their performed length can be verified in Tab. 5.

**Table 5.** Micropile Data

Frame Axis	N°of Micropiles	L (m)	Frame Axis	N°of Micropiles	L (m)
1	4	26	8	4	19
2	4	23	9	4	16
3	6	22	10	4	14
4	4	21	11	4	13
5	4	21	12	4	14
6	4	19	13	4	18
7	8	19	14	4	13



**Figure 4.** Geotechnical model and SPT's, Frame Axis' and Static Load Test location

The reliability analysis evaluates in this paper focuses on the micropiles situated on the frame axis number 9 as highlighted on the previous table. The reason behind that is the conduction of a static load test in one of the micropile of this frame axis which will serve as verification of the reliability analysis (Fig. 4).

#### 4. Reliability Analysis

To verify the reliability of the performed micropile ( $\phi=0.31$  m and  $L=16$  m) based on the formulation described in Eq. 14 it was necessary to follow some previous procedures.

Firstly, it was necessary to define the random variables ( $N_L$  and  $D$ ) and their respective statistical parameters ( $\mu$ ,  $SD$ ,  $CV$ ,  $PDF$  and  $CDF$ ). For  $N_L$  values, the tests SPD1, SPE1 and SP1 to SP6 were considered to study its variability. The values of the tests from SP7 to SP10 were not used due to the presence of an unrepresentative soft black clay layer in the scenario under analysis (single micropile bearing capacity on frame axis A-09). This means that these Standard Penetration Tests do not represent the geotechnical model of the micropiles performed in the bridge frame axis A-09 (Fig.4). Therefore, the  $N_L$  value of each SPT considered in the geotechnical model was calculated to obtain its statistical parameters (Tab. 6). In terms of Demand ( $D$ ) variability, the values from the foundation project were considered ( $\mu_D = 800$  kN;  $CV_D = 0.1$ ; normally distributed PDF and CDF).

Table 6. Parameters of variables

MICROPILE ( $\phi = 0.31$ m and $L = 16$ m)		
SPT	$N_L$	$D$ (kN)
SPD1	25	-
SPE1	22	-
SP1	22	-
SP2	22	-
SP3	19	-
SP4	20	-
SP5	17	-
SP6	15	-
$\mu_{N_L}$	20	800
$SD_{N_L}$	3	80
$CV_{N_L}$	16%	0.1

Secondly, in order to verify if the set of  $N_L$  values follows a normaly distribution function, the one-sample Kolmogorov–Smirnov test was conducted with the support of MATLAB’s function kstest. The test returns two values of  $h$ : 1 or 0. If the test returns a  $h$  value of 1 the test fails, otherwise it succeeds. The kstest also return the level of agreement between the CDF’s functions ( $p$ -value). As closer this value is to 1.0, a better fit the curves have. For this scenario, the  $h$  value was equal to 0 and the  $p$ -value was equal to 0.8144, which means that is reasonable to affirm that  $N_L$  values follows a normaly distributed function. The Fig. 5 describes the Standard CDF and the Empirical CDF together after the kstest.

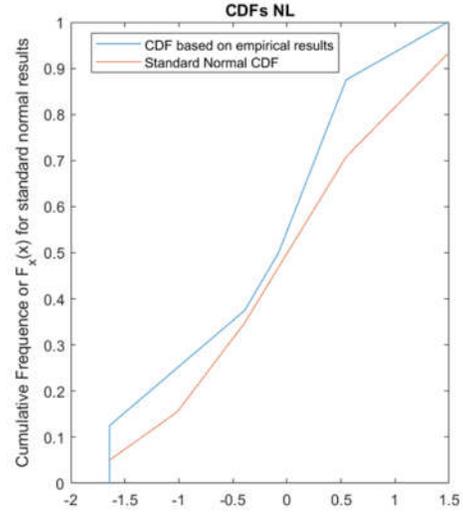


Figure 5. Empirical and Standard Normal CDF

Finally, the reliability analysis were conducted with FOSM and MCM methodology.

The FOSM reliability index was calculated as it follows:

1. The limit state function for the single micropile bearing capacity presented in this paper is equal to Eq. 17;
2. Substituting the for  $\beta$ ,  $U$  and  $L$ , the performance function can be rewritten as (Eq. 18, 19):

$$g(X_i) = g(N_s, S) = 1.5 * 10 * \left(\frac{N_s}{3} + 1\right) * 0.9739 * 16 - D \quad (18)$$

$$g(N_s, S) = 233.736 + 77.912 * N_L - D \quad (19)$$

3. Since the performace function is linear, Eq. 11 can be used to calculate the reliability index:

$$\beta_{FOSM} = \frac{233.736 + (77.912 * 20) - (1 * 800)}{\sqrt{[(77.912 * 3)^2 + (-1 * 80)^2]}} = 4.01$$

4. As the random variables of the performance functions are normaly distributed, the relation expressed in Eq. 2 is valid. Therefore the probability of failure is:

$$p_{fFOSM} = \Phi(-4.1) = 1 - \Phi(4.1) = 3 * 10^{-5}$$

The Monte Carlo Method reliability index was calculated using the software MATLAB and the methodology described in the previous section (2.4) with  $n$  (number of simulations) equal to 100.000. The reached reliability index was  $\beta_{MCM} = 3.719$  and a related probability of failure of  $p_{fMCM} = 1 * 10^{-4}$  with a number of collapses  $n_c$  equal to 12. The output graphs as histogram, PDF, empirical CDF and the 3D representation of the performance function can be observed on the Fig. 6 to 9.

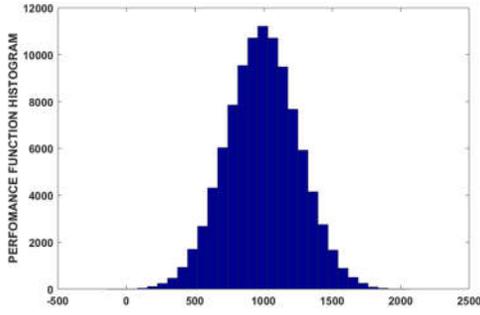


Figure 6. Performance function histogram

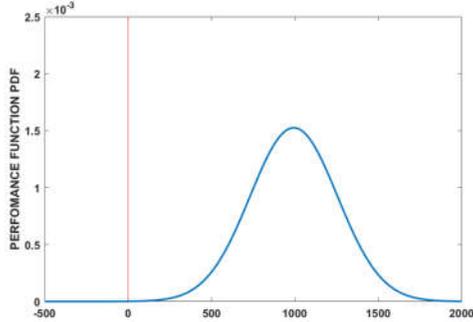


Figure 7. Performance function PDF

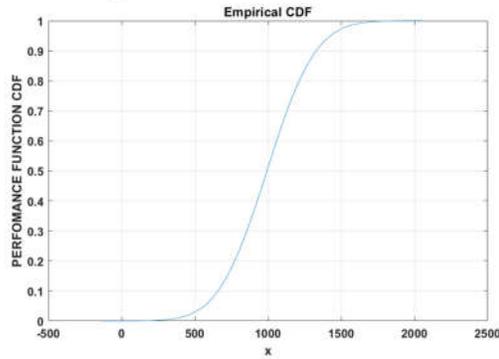


Figure 8. Performance function CDF

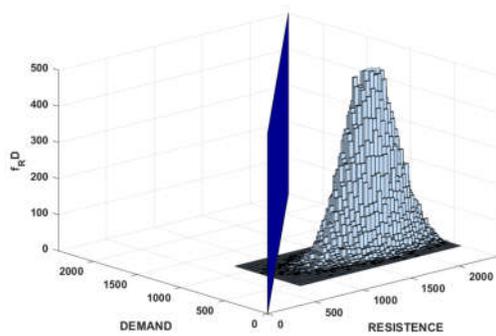


Figure 9. 3D visualization of the performance function

The factor of safety (FOS) of this scenario considering the average values into the resistance function instead of a function of normally distributed random variables is:

$$FOS = \frac{1.5 * 10 * \left(\frac{20}{3} + 1\right) * 0.9739 * 16}{800} = 2.2$$

Lastly, the reliability analysis (FOSM and MCM) changing micropiles' lengths from 15 meters to 10 meters was performed to analyze its behaviour. A summary of the results can be observed in Tab. 7 and the events description in Tab. 8. The curves in Fig. 10 also represent the average values of the safety parameters behaviour.

Table 7. Reliability analysis with decreasing micropile length

Length (m)	16	15	14	13	12	11	10
kstest	0	0	0	0	0	0	0
p-value	0.81	0.90	0.98	0.87	0.84	0.6	0.84
DV(kN)	1791	1636	1492	1342	1202	1068	941
FOS	2.2	2.0	1.9	1.7	1.5	1.3	1.2
$\beta_{FOSM}$	4.01	3.46	3.18	2.57	2.3	1.5	1.29
$p_{FOSM}$	$10^{-5}$	$10^{-4}$	$10^{-4}$	$10^{-3}$	$10^{-2}$	$10^{-2}$	$10^{-1}$
$\beta_{MCM}$	3.67	3.63	3.33	2.83	2.28	1.63	1.16
$p_{MCM}$	$10^{-4}$	$10^{-4}$	$10^{-4}$	$10^{-3}$	$10^{-2}$	$10^{-2}$	$10^{-1}$
$p_{collapses}$	12	14	43	255	1075	5161	12348

\*DV = Deterministic Value of Bearing Capacity; FOS = Factor of Safety;  $p_{collapses}$  = Number of Collapses in the Simulation

Table 8. Description based on the probability scale by [10]

Length (m)	$\beta_{FOSM}$	Description FOSM	$\beta_{MCM}$	Description MCM
16	4.01	Improbable	3.67	Improbable
15	3.46	Remote	3.63	Remote
14	3.18	Remote	3.33	Remote
13	2.57	Ocasional	2.83	Ocasional
12	2.3	Ocasional	2.28	Ocasional
11	1.5	Probable	1.63	Probable
10	1.29	Probable	1.16	Probable

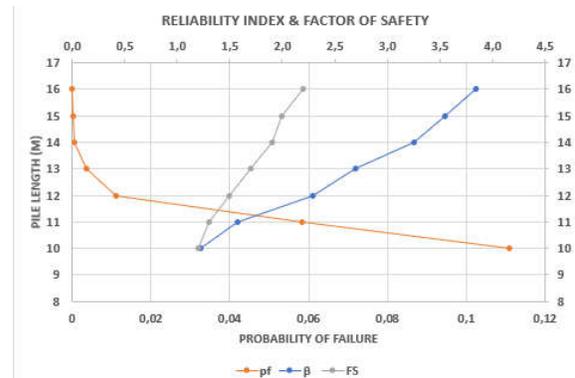


Figure 10. Behaviour of  $\beta$ ,  $p_f$  and FOS through pile's length

For each length scenario it was plotted the PDF, the CDF and the 3D visualization of the performance functions (Fig. 11 to 16).

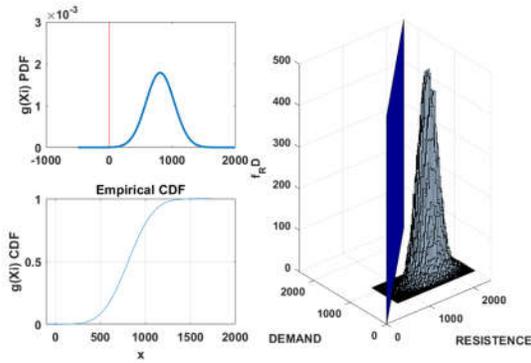


Figure 11. PDF, CDF and 3D visualization for  $L = 15\text{m}$

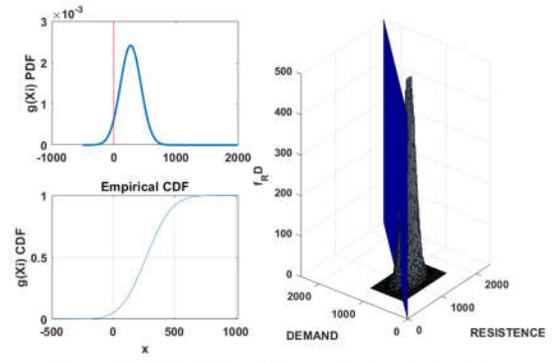


Figure 15. PDF, CDF and 3D visualization for  $L = 11\text{m}$

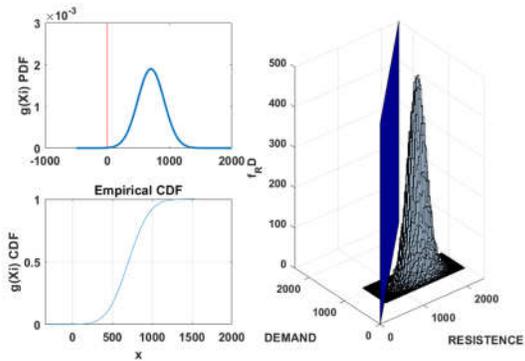


Figure 12. PDF, CDF and 3D visualization for  $L = 14\text{m}$

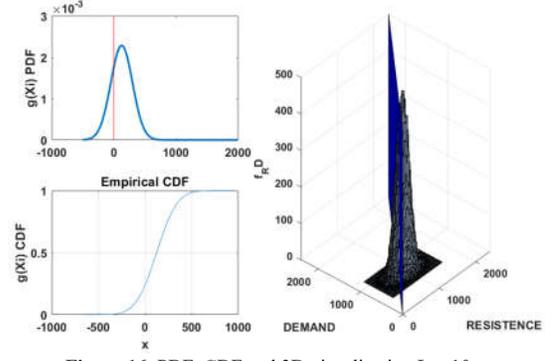


Figure 16. PDF, CDF and 3D visualization  $L = 10\text{m}$

## 5. Mixed Performance Static Load Test

A mixed static load test was performed to verify the foundation's performance after construction. According to [24], firstly, the test is conducted as a slow static load test until the load value of twenty percent higher than the workload estimated for the pile. It means that the load stage shall be carried out in equal and successive stages, observing that (1) the load applied at each stage must not exceed 20% of the expected workload for the tested pile and (2) at each stage the load must be maintained until the stabilization of pile's settlements and for at least 30 minutes.

Secondly, after the load reached a 20% value higher of the estimated workload, the static load test shall be conducted as a rapid static load test. In other words, the load stage shall be carried out in equal and successive stages, noting that (1) the load applied at each stage shall not exceed 10% of the expected workload for the tested pile and (2) at each stage the load shall be maintained for 5 minutes, whether or not pile's settlement stabilized.

For the tested single micropile ( $\phi = 0.31\text{m}$  and  $L = 16\text{m}$ ), the calculation of the deterministic geotechnical load capacity, according to the exposed method, was 1791 kN. As the structural capacity of this micropile is of the same magnitude, the test was limited to a load of 1.800 kN for safety reasons. The tested micropile showed no rupture at the pile-ground interface, reaching a maximum load of 1.544 kN with a total displacement of 2.8 mm. It can be noted in Fig. 17 that the behavior of the foundation element was essentially elastic and the total displacement for a deterministic workload scenario ( $D = 800\text{ kN}$ ) is around 1.2 mm.

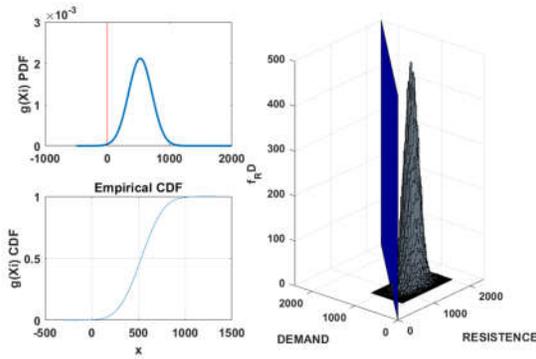


Figure 13. PDF, CDF and 3D visualization for  $L = 13\text{m}$

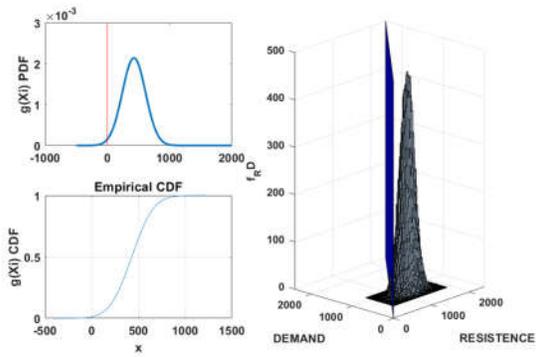


Figure 14. PDF, CDF and 3D visualization for  $L = 12\text{m}$

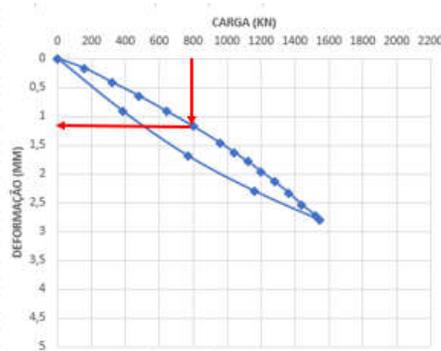


Figure 17. Mixed Performance Static Load Test

## 6. General Results

From the reliability analysis, it can be affirmed that the micropiles constructed in the bridge frame axis number nine have a probability of failure value near zero (around  $10^{-5}$ ) and a high reliability index (around 4.0). In addition, the safety factor follows that established by [12], i.e. a value of 2.0 or higher since no preload tests were performed in order to optimize its length.

In order to verify the effect of length variation on the pile reliability index in this section of the viaduct, a series of simulations were made by reducing the micropile's length until the value of 10 meters. It was observed that for length values of 10 and 11 meters the reliability index presented high  $p_f$  values. For a hypothetical 12-meter length micropile, a  $10^{-2}$  probability failure was found which indicated a occasional risk of failure. However, as the micropiles will work together (two-pile blocks interconnected by themselves) and not as single piles, it has been found that the reliability index values  $\beta$  for a 12 meters micropiles are consistent with the proposal of [11] for a reliable foundation. Finally, the results shown by piles of theoretical lengths from 13 to 15 meters proved to be reliable for an isolated foundation element as proposed by [11], with  $\beta$  values between 2.6 and 3.6 associated with a failure probability between  $10^{-3}$  and  $10^{-4}$ .

It is noteworthy that for all reliability scenarios, the random lateral resistance variable ( $N_l$ ) behaved reasonably linearly ( $p$ -value between 0.8 and 1.0), except for the pile length scenario of  $L = 11$  meters ( $p$ -value = 0,6).

Lastly, it can be observed that the pile behaved in an elastic way regarding the mix performance static load test, not reaching the geotechnical rupture (total displacement values below 10% of the pile diameter).

## 7. Conclusions

This paper describes the application of two reliability-based methodologies applied in a specific case study of axially loaded single micropile. This work was performed to be used as an aid to pile engineer designers in assessing the uncertainties associated with the random variables that most influence the behaviour of micropiles bearing capacity performance function based on in situ SPT results.

By considering the inherent soil variability in the construction site through the Standard Penetration Test results, the following notes can be made:

- The FOSM and MCM methodologies presented a good agreement with the reality, mostly because their boundary conditions as the use of a linear performance function for FOSM analyses and the knowledge of the random variable's type of distribution for MCM analyses were both satisfied;
- The reliability analyses for the actual pile length ( $L=16$  m) demonstrated conservative values in compare with the values from the literature;
- The reliability simulations with pile lengths' varying from 13 meters to 15 meters demonstrated safety and reliable values of  $\beta$  and  $p_f$  which would lead to a cost-effective design when compared with the actual length.
- The mixed load test was satisfactory, reaching a axial capacity equal to twice the workload without evidence of geotechnical failure between the soil-pile interface. It can also be concluded that the results of load test had good agreement with the reliability analyses of the performed micropiles length (conservative values);
- If the load test were performed before the construction of the pile, i.e. in the design phase, the factor of safety to be applied could be equal to 1.6 according to the Brazilian standard. It implies that the probable piles' length to support the demand on the pile would be between 13 and 14 meters which indicates a concur with the overall reliability analyses done in this paper for a reliable micropile.

In summary, the results of the reliability analysis of pile foundations to understand the variability of resistance and demand conditions is a powerful tool for assessing safety and mitigating future accidents. Therefore, the practice of this technique in foundation engineering should be encouraged in order to provide more realistic information about the uncertainties present in this sphere.

## Acknowledgement

The authors gratefully acknowledge the Brazilian Agency CAPES (Brazilian Federal Agency for Support and Evaluation of Graduate Education) for the scholarship conceived.

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